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Recurring angular distribution patterns in resonant heavy-ion reactions

R.M. Freeman[1], F. Haas[1], M.-P. Nicoli[1], A. Morsad[2], Z. Basrak[3]

[1] Institut de Recherches Subatomiques, IN2P3-CNRS / Université Louis Pasteur (UMR 7500) B.P.28, F-67037 Strasbourg Cedex 2, France
[2] Faculté de Sciences Ben Msik, Université Hassan II, Casablanca, Morocco
[3] Ruđer Bošković Institute, Zagreb, Croatia

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Abstract. It has been noticed that a distinct resemblance exists at large angles among angular distributions measured for reaction channels at the energies of some resonance-like structures in the ^{12}C+^{12}C and ^{14}C+^{16}O systems. It is pointed out that such forms are typical of a diffraction pattern of a broad band of coherent partial waves.

PACS. 25.70.Ef Resonances – 25.70.Hi Transfer reactions

1 Introduction

The angular distribution measured by the Argonne group [1,2] for the mutual excitation of the second 0^+ state of ^{12}C on a resonance of the ^{12}C+^{12}C reaction has aroused considerable interest. The strong constructive interference which is observed around 90^o indicates interference between a number of partial waves and lends support to their claim that a chain state is excited in the compound nucleus ^{24}Mg. We have since pointed out [3] that the $^{8}Be_{g.s.}$+$^{16}O_{g.s.}$ outgoing channel also resonates at this energy, an observation which is not directly reconcilable with the idea of a chain state. On the other hand we noted the similarity at large angles between the angular distribution for this channel and that published by the Argonne group. To best appreciate the resemblance between the two results the angular distribution of the Catania group [4], who have measured the on-resonance angular distribution of the ^{8}Be+^{16}O channel over a wide angular range, should be compared with the more recent data of the Argonne group [2] for the mutual inelastic scattering. Both sets of data are characterized by the same modulated pattern where the constructive interference at 90^o is accompanied by low cross sections in the region of 70^o. This modulation has a cycle of about 45^o and in the next section we will interpret it as a beating between partial waves with quite different l values.

More recently we have discovered that this type of angular distribution occurs for resonances in the heavy-ion reaction ^{14}C+^{16}O. Resonances have been previously observed in the γ-ray yields [5] of this reaction, particularly for transitions from the excited states of ^{18}O where the yields show the most prominent maxima at energies $E_{c.m.}$=23.5 and 27.5 MeV. Later we studied channels of this reaction which are undetected by the γ-ray method using particle techniques [6] and it was on inspecting a part of these data that had not previously been published that we came upon angular distributions in the $^{15}N_{g.s.}$+$^{15}N_{g.s.}$ outgoing channel which are modulated in a similar manner at the resonant energies. Like the ^{12}C+^{12}C reaction this channel involves identical particles with the consequence that the distributions are symmetric about 90^o. Unlike the spin 0 boson ^{12}C, the fermion ^{15}N has spin 1/2 which complicates the description of the reaction channel. However, it has been observed that the even l partial waves dominate in the resonant process. In this case it is the antisymmetric S=0 spin state which is appropriate for the ^{15}N+^{15}N channel. This is probably why the shapes of the angular distributions which are experimentally observed on-resonance are similar whether they are for identical bosons or identical fermions.

In Fig. 1 we compare the angular distributions at angles beyond 50^o for two energies of the $^{16}O(^{14}C,\ ^{15}N)^{15}N$ reaction with the data of the Catania group for the $^{12}C(^{12}C,\ ^{8}Be)^{16}O$ reaction. Over the angular range displayed in this figure the similarity between the three distributions is evident; the cross sections are large at 90^o but dip down to low values near 70^o. All three angular distributions were measured at the summit of a heavy-ion resonance. The data of the Catania group [4] were recorded at the $E_{c.m.}$=32.5 MeV resonance of the ^{12}C+^{12}C reaction first studied by the Argonne group. The other two we have measured at energies which correspond to known

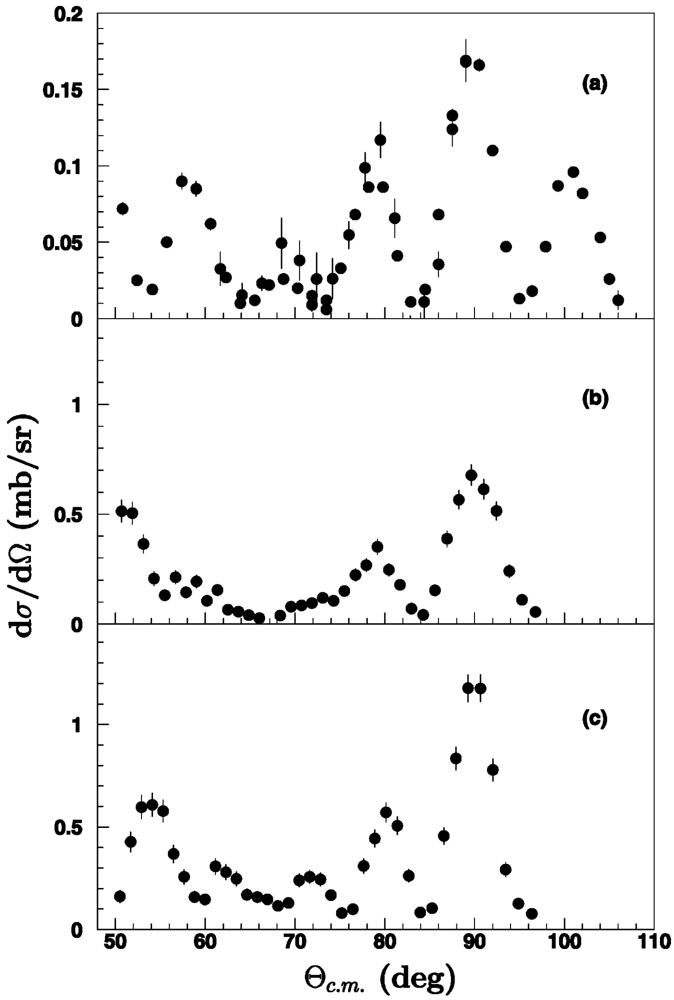

Fig. 1. On-resonance angular distributions for symmetric reactions; (a) the $^{12}C(^{12}C,\ ^{8}Be_{g.s.})^{16}O_{g.s.}$ reaction on the $E_{c.m.}$=32.5 MeV resonance, (b) the $^{16}O(^{14}C,\ ^{15}N_{g.s.})^{15}N_{g.s.}$ reaction on the $E_{c.m.}$=23.4 MeV resonance, (c) as for (b) but on the 27.4 MeV resonance

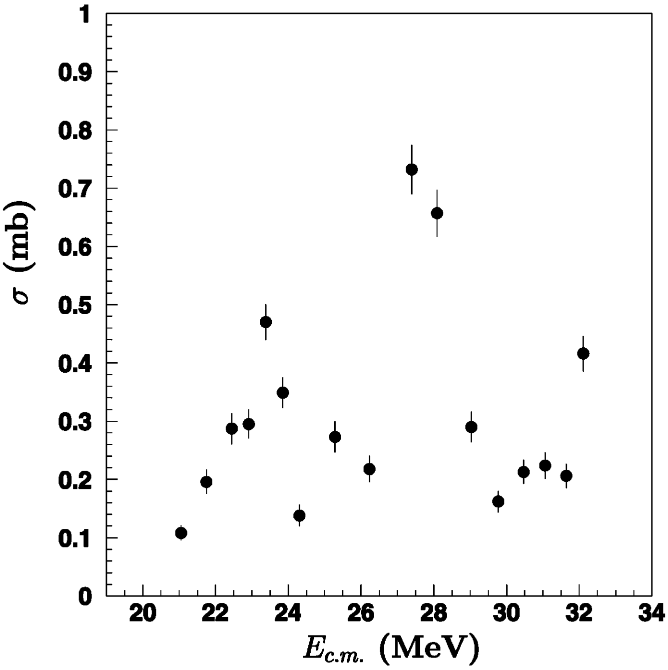

Fig. 2. Excitation function of the $^{16}O(^{14}C,\ ^{15}N_{g.s.})^{15}N_{g.s.}$ reaction integrated over the angular range 85^o-95^o

resonances in the ^{14}C+^{16}O system [5]. Away from these resonances the angular distributions are quite different.

An excitation function for the $^{15}N_{g.s.}$+$^{15}N_{g.s.}$ channel, derived from our ^{14}C+^{16}O data, is shown in Fig. 2. The two resonances stand out particularly clearly in this figure as the intensities were determined for angles around 90^o where the structure is heightened by the effects of the constructive interference.

It is curious that for these reactions where, in addition to being symmetric, the description of the channel is less complex due to the low (0 or 1/2) spin of the fragments the same type of angular distribution is observed on the summit of the resonances. Obviously some common explanation is called for. In the next section we will describe the analysis which we have made of the angular distributions. A simple recipe will be proposed which, when applied to the $^{12}C(^{12}C,\ ^{8}Be)^{16}O$ data, leads to the right form for the angular distribution and, with a fitting procedure, directly to convergence onto the experimental data.

2 Analysis of the angular distributions

For two-body reactions involving only spin 0 nuclei, which will be the case for the example treated in this section, the differential cross section as a function of angle reduces to the expression:

$$d\sigma/d\Omega = (1/4k^2)|\Sigma_l S_l(2l+1)P_l(cos\theta)|^2 \qquad (1)$$

where the S_l for each partial wave l can be written in terms of an amplitude and phase in the following manner:

$$S_l = a_l e^{i\phi_l} \qquad (2)$$

The channel cross section for a partial wave is given by:

$$\sigma_l = (\pi/k^2)(2l+1)a_l^2 \qquad (3)$$

where $a_l^2 \leq 1.0$. This is further simplified in the case of the identical boson reaction ^{12}C+^{12}C by summing over only the even l partial waves. In this case the expression given by (1) and (3) should be multiplied by 4 assuming that no distinction is made between the outgoing particles. For the ^{8}Be+^{16}O channel the distinction is made so the factor should be reduced to 2. The structure in the angular distributions tends to be washed out if the phase shifts for the main partial waves contributing to the reaction are very different from one another. As the experimental angular distributions are highly structured we begin the analyses with phases for the initial parameters which are equal or nearly equal.

We will consider the angular distribution data of the Catania group on the $E_{c.m.}$=32.5 MeV resonance of ^{12}C+^{12}C, part of which is shown in Fig. 1(a). The high

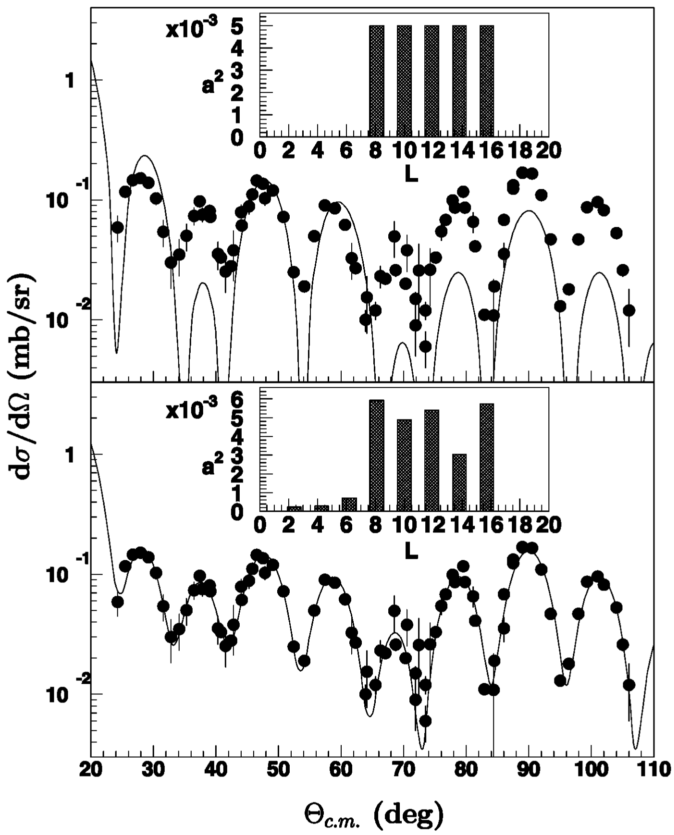

Fig. 3. Fits to the experimental angular distribution of Ref.4 on the $E_{c.m.}$=32.5 MeV resonance of the $^{12}C(^{12}C, {}^{8}Be_{g.s.})^{16}O_{g.s.}$ reaction. The upper part shows a one-parameter fit as described in the text. The lower part shows the full 19-parameter (10 amplitudes and 9 phases) fit

statistics and large angular range of these data, as well as the spin 0 condition, make it the best candidate for a partial wave analysis. As mentioned in the previous section a modulation with a 45^o cycle appears in the angular distribution. A beating between partial waves differing by 8 units of angular momentum ($\Delta l=2\pi/\Delta\theta=360^o/45^o$) would produce such an effect. The main frequency in this angular distribution corresponds to an l=16 partial wave. Interference between l=16 and 8 would therefore reproduce the modulation observed in the data. In fact a good fit is obtained to the experimental data with just the simple beating between these two partial waves for angles from 60^o to 90^o. With a band of partial waves between these two l values the form is not dissimilar, especially at the larger angles, but it agrees even better with the experimental data at more forward angles.

In Fig. 3 the angular distribution is shown over the whole experimental range. The curves which correspond to the fit with the starting set of parameters and with the final conditions after parameter ajustment are shown in the upper and lower parts of this figure, respectively. The similarity between the angular distributions of Fig. 1 is hardly likely to be the outcome of some haphazard combination of parameters so for the starting conditions we have chosen the simplest possible set of parameters for a band of partial waves; for all l values from 8 to 16 the amplitudes were set to the same value, the same sign and with the same phase. The initial value of the amplitude is the only free parameter in this fit. The resulting fit, which is shown in the upper part of Fig. 3, already bears a strong resemblance to the experimental data. From these initial conditions it is relatively easy to home onto a set of parameters which fit perfectly as displayed in the lower part of the figure. The price to pay was the introduction of 18 additional parameters (all even partial waves from l=0 to 18 were included though the amplitudes for the extreme l values are too weak to be visible in the histogram) without fundamentally changing the band structure of our original set. The phases within the band varied little and the final values remained relatively close to each other.

Technically the fitting proceeded as follows. A χ^2 was defined as a criterion of the quality of the fit. This χ^2 was first minimized for a linear series of Legendre polynomials from which we obtained the maximum l value which should be included in the analysis and the minimum value of the χ^2 which we could hope to achieve. It also provided us with an approximate value for the cross section at 0^o which was included with a large error bar in the experimental results to anchor the forward angle cross sections from inflating to unreasonable values during the fitting procedure; the 'blow-up' effect known to those who attempt such analyses. Then the derivation of the χ^2 with respect to the parameters was determined and the parameters were changed accordingly to minimize the χ^2. The amplitudes and the phases were varied separately and alternatively until a fit of the quality seen in Fig. 3 was obtained.

Little importance can be attributed to the precise values that were finally obtained for the parameters. There are 256 ($2^{l_{max}/2-1}$) independant sets of parameters which produce exactly the same curve [7] displayed in Fig. 3 though a similar band structure for the amplitudes is omnipresent among all these other possibilities. In addition we have not considered waves beyond l=18 though they should be there to some extent. What is significant in Fig. 3 is that although other parameter sets which include the l=8 and 16 partial waves may be found which give a reasonable fit to the experimental data the simplest already gives the right pattern for the angular distributions before any adjustment of the values. We may never know what are the exact values of the parameters but the type of parameter set proposed explains naturally the recurrent shapes in Fig. 1.

Another quite different set of parameters was investigated which gives an equally good fit to the data and is supposedly of the same family of parameters as that proposed by the Catania group [8]. The magnitude of the amplitudes for this set is greatest for l=0 and decreases towards zero for the grazing angular momentum. The fitting procedure tends to drive the parameters towards this set if too much liberty is allowed in the way they can change. Large intensities are obtained for the outgoing low l partial waves which approach 40% of unitarity. Even if one

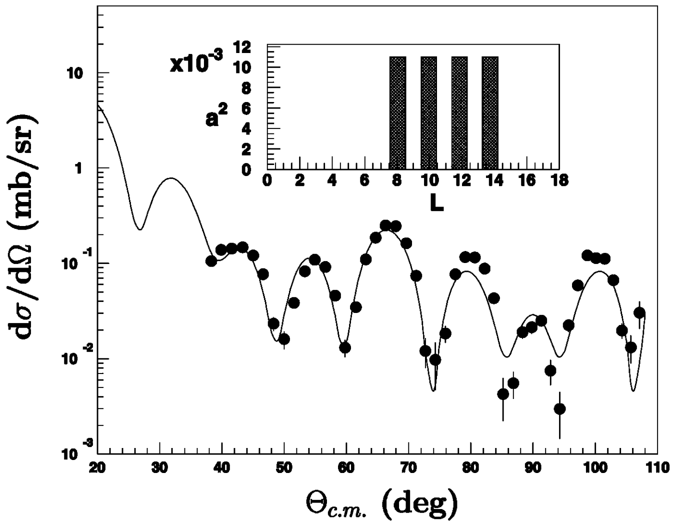

Fig. 4. Angular distribution for the $^{12}C(^{12}C,\ ^{8}Be_{g.s.})^{16}O_{g.s.}$ reaction at $E_{c.m.}$=27.5 MeV. Only the phases in the partial wave analysis were varied for the fit as described in the text; the amplitudes were held constant at the values shown in the histogram

accepts the possibility of such intensities there is another reason for rejecting this type of fit. Parameters of this set can generate a whole series of forms and it is necessary to tune the amplitudes to obtain the desired shape of the angular distribution unlike the fit in Fig. 3 where the right shape is present from the outset and remains relatively stable under modification of the parameters. It is evident that, to explain the constancy of the forms of Fig. 1, this stability should be a property of the parameter set.

Ideally we would have liked to have extended this analysis to other energies to follow the evolution of the intensities of the partial waves before and beyond the resonance but in view of the large number of possible solutions this does not appear to be feasible. However, looking at other angular distributions of the Catania group, most of which have not been published, some remarks can be made about how they evolve at lower bombarding energies. In the region around $E_{c.m.}$=28.5 MeV the angular distributions [9] take on a more regular form closer to the shape for a pure l=14 partial wave at least at large angles (the experimental data cover a smaller angular range than at 32.5 MeV). Other waves are certainly present but there is no salient indication in the data of what these waves might be. Lower still near $E_{c.m.}$=27.5 MeV the modulated shapes become more apparent again though inverted with respect to the forms shown in Fig. 1. Such forms are easier to fit because the modulation allows physical insight into what should be introduced into the partial wave analysis. One of the angular distributions in this region is shown in Fig. 4 and is presented here as an example of the other possible type of pattern which can be expected from a coherent band of partial waves. Angular distributions with a large maximum at 90^o as in Fig. 1 or a small maximum as in Fig. 4 can be simulated with either an odd or even number of partial waves in the band, respectively. This behavior can be traced to the sign change of the Legendre polynomials at 90^o

$$P_{l+2}(0) = -P_l(0)(l+1)/(l+2) \tag{4}$$

as l is incremented by 2. There could be situations where an alternation between the two possible forms would be observed. This is exactly what is seen in the data between $E_{c.m.}$=32.5 and 27.5 MeV where the grazing angular momentum changes by approximately two units.

A more restricted type of fit was attempted for this angular distribution as the data cover a narrower angular range. The starting point for the fit was similar to that in Fig. 3 except that the l=16 partial wave was suppressed. Then with a set of identical and fixed amplitudes for l=8 to 14 normalized to fit roughly the experimental data we varied only the phases. The most evident effect of varying the phases is to attenuate the angular structure (experimental angular resolutions can also damp the structure). In this very simple way we have obtained an acceptable fit to the data as shown in Fig. 4. The maximum difference in phase among the four partial waves is 35^o.

Whereas only weak structures in the excitation functions [10] have been reported in the energy region of the data of Fig. 4. a prominent structure in the inelastic channel to the 0_2^+ state of the ^{12}C+^{12}C reaction has been observed at a neighbouring energy. This structure at $E_{c.m.}$=29.0 MeV was first studied by Fulton, Cormier and Herman [11]. Pate et al. [12] have measured angular distributions out to beyond 90^o for this reaction at a number of energies in the vicinity of this structure. On the summit at $E_{c.m.}$=29.0 Mev a similar angular distribution to that shown in Fig. 4 is seen. They were unable to fit their distributions adequately with partial waves down to l=12. Their analysis was critizied by Lindsay [13] for not including lower l partial waves and in the light of our experience this criticism is justified. The shape of the angular distribution demands waves down to l=8 to account for the modulation as in Fig. 4.

3 Discussion

It was shown in the previous section how good fits can be obtained for angular distributions of the type presented in Fig. 1 and Fig. 4. Though no unique set of parameters can be retained from these analyses this should not detract from the conclusion that amplitudes of comparable magnitude for partial waves over a surprisingly wide range of l values are required to fit the angular distributions. Heavy-ion reactions at these energies are regarded as strongly absorbing. The nuclei appear black except in the penumbral zone around the grazing l value. For example, the ^{12}C+^{12}C reaction has a fusion cross section of about 700 mb at $E_{c.m.}$=32.5 MeV corresponding to grazing and critical angular momenta of l=16 and 14 respectively. Therefore all waves from the interior out to about l=14 should be strongly absorbed. There is no evidence of this in the experimental data; waves from the interior as strong as the grazing partial wave are needed to fit the form of the angular distributions.

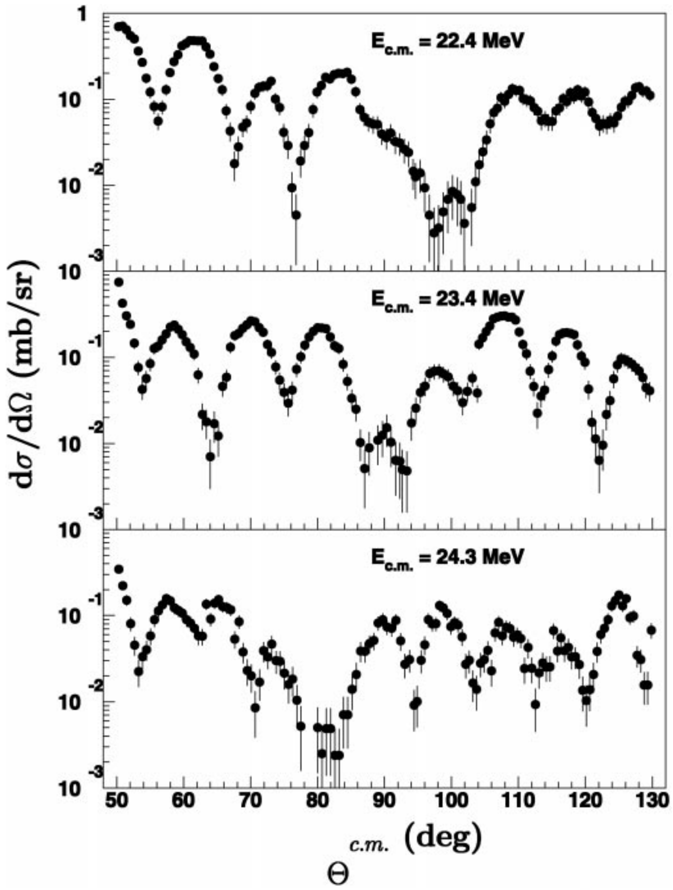

Fig. 5. Evolution of the angular distributions for an asymmetric channel across the first resonance of Fig. 2. The experimental data were recorded for the $^{12}C(^{18}O,\, ^{16}O_{g.s.})^{14}C_{g.s.}$ reaction but the energies are given for the inverse reaction (+0.9 MeV) to be compatible with Fig. 2

To further illustrate how important are the contributions from lower l values we present in Fig. 5 results for another channel of the ^{14}C+^{16}O reaction. The evolution of the angular distributions of the $^{12}C_{g.s.}$+$^{18}O_{g.s.}$ channel can be seen as the energies cross the first resonant structure of Fig. 2. In reality we show the data for the inverse reaction where our result cover a wider angular range but the energies which are given are for the ^{14}C+^{16}O entrance channel to facilitate comparison with Fig. 2. Some of these data have been published previously [6]. Unlike the examples that have been considered up to now this reaction is not symmetric about 90^o and consequently both even and odd partial waves must be included in the analysis. A hole appears in the angular distributions which moves with increasing energy to smaller angles crossing 90^o at the peak of the resonance. (Note that the direction of the angular scale is arbitary depending on which of the two outgoing fragments was chosen as reference.) At higher energies still, close to the second resonance, the hole reappears at about 110^o but thereafter the behaviour becomes unclear probably because the second resonance is not very prominent in this channel.

In view of the relative ease with which good fits were made to data in a symmetric channel as in Fig. 3 it could be anticipated that a similar approach should succeed here. A previous attempt had been made to fit these data as described in our earlier work. In the spirit of the analysis that we attempted there, the fitting procedure started from a set of parameters where the amplitudes of the partial waves were greatest around the grazing angular momentum. The resulting fits were too regular to resemble the experimental data and an l=0 wave was added, rather arbitarily, to help break this regularity. Holes like those seen in Fig. 5 could not be reproduced and the example of a fit which we presented in that article was for an angular distribution at an energy where no hole was apparent. The situation changes using a band of partial waves and curves, where there is some resemblance to the experimental data are found. The patterns move across 90^o as new partial waves are added to the band simulating the evolution of the angular distributions displayed in Fig. 5.

However we never succeeded in obtaining angular distributions close enough to the experimental forms to carry through this analysis. A band of partial waves of equal amplitude gives a repetitive pattern across the entire angular range whereas the hole, which is the conspicuous feature in the experimental data of Fig. 5, was restricted to a limited zone around 90^o. A similar situation can be seen in Fig. 3, though to a much less severe degree, where the initial fit shows a strongly repetitive pattern whereas the beating in the experimental results tends to fade out towards the more forward angles. Our failure to converge on a closer fit can in part be also ascribed to the narrower angular range of the experimental data. Therefore we have not shown any fits in Fig. 5 but instead will insist on what can be more reliably claimed, i.e. that such forms for the angular distributions can only be explained by including widely different l-values in the partial wave analysis. In this respect the progression of a low minimum moving across angular distributions with changing incident energy can be likened to the behaviour of the Airy minima observed in elastic scattering (see Ref.14 for example). What is common to these features in the data of the reaction channel of Fig. 5 and the Airy minima in elastic scattering is that highly penetrating interior waves are required to fit the experimental angular distributions.

There is nothing new about how the interference between many partial waves can show up strongly in the angular distributions of some light heavy-ion reactions like ^{12}C+^{12}C, in spite of the strong absorption which is evidently present. More than twenty years ago Rowley, Doubre and Marty argued that low l partial waves [15] were necessary to explain the ^{12}C+^{12}C elastic cross sections at 90^o. Others have also found that low l partial waves are important in the description of this system. For example, at $E_{c.m.}$=37.1 MeV, an energy somewhat higher than that of Fig. 3, McVoy and Brandan report beating between l=18 and 6 partial waves in the elastic angular distributions [14]. As we have already discussed Lindsay [13] criticized the analysis of Pate et al. of their experimental data [12] for the inelastic channel to the 0_2^+ state on the grounds that they had not included low l partial waves. There is evidence that a wider range of l values should have been introduced to fit their angular distributions. The Argonne group remarked on the intensity of their angular distributions at 90^o as if a chain state with high moment of inertia could be responsible for the constructive interference of several partial waves at this angle. This has been expressed more explicitly by Rae, Merchant and Buck [16]. Our analysis is not based on any model. We have just drawn attention to the possibility that the angular distributions can be explained with a coherent band of partial waves where the modulation results from the low l cut-off of the band.

What is more noteworthy in the present article is that we have shown that angular distributions of the type observed on the $E_{c.m.}$=32.5 MeV resonance of the $^{12}C+^{12}C$ reaction are not isolated examples but they occur also at resonant energies of $^{14}C+^{16}O$ implying that what is at the origin is more general than a specific effect for $^{12}C+^{12}C$. We have demonstrated that these experimental data show features which are characteristic of the interference within a wide band of partial waves. For a number of cases the angular distributions which are observed, notably at the summit of resonance-like structures, resemble what could be expected from the diffraction pattern of a broad ring which is uniformly illuminated.

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